Opener

The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

28 8 12 -8

(A) 4 (B) 2 ~~(C) 1~~ (D) 0 (E) −2

candidates

endpts: x = -2, 4

der = 0: x = 0, 2

der = undef: N/A

$f' = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$= x(3x - 6$

$x = 0 \quad 3x - 6 = 0$

$x = 2$

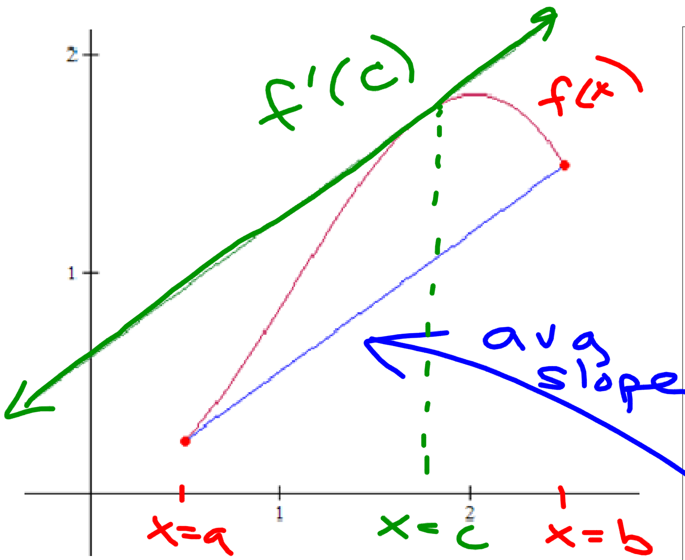
4-2 Mean Value Theorem

Learning Objectives:

I can apply the Mean Value Theorem to find a location for which the instantaneous slope equals the average slope.

I can identify when a function is increasing and when it is decreasing and I understanding the relationship between this and the derivative of the function.

I can find the antiderivative of a function.



Mean Value Theorem (Part 1)

If y = f(x) is continuous at every point in the closed interval [a,b] and differentiable at every point in the open interval (a,b), then there is at least one point c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Ex1. Show that each function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find a solution "c" to the equation $f'(c) = \frac{f(b)-f(a)}{b-a}$

avg slope

MVT applies

1.) $f(x) = 3x^2 + 2x + 5$ on [-1,3]

(-1, 6)
(3, 38)

avg slope $= \frac{38-6}{3+1} = \frac{32}{4} = 8$

$f' = 6x + 2$

$8 = 6x + 2$

$6 = 6x$ $x = 1$

MVT applies

2.) $f(x)=\sin(2x)$ on $\left[0,\frac{\pi}{4}\right]$

$(0,0)$

$\left(\frac{\pi}{4},1\right)$

$\frac{1}{\frac{\pi}{4}}=\frac{4}{\pi}$ ← avg slope

$f'(x)=2\cos(2x)$

$\frac{\frac{4}{\pi}}{2}=\frac{2\cos(2x)}{2}$

$\cos 2x=\frac{2}{\pi}$

$2x=\cos^{-1}\frac{2}{\pi}$

$x=\frac{\cos^{-1}\frac{2}{\pi}}{2}$

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3.) $f(x)=x^{2/3}$ on [-1,8]

NO MVT
does not apply
cusp @ $x=0$

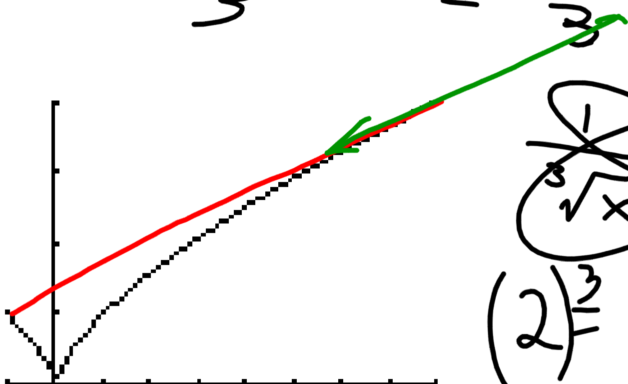
$(-1, 1)$

$(8, 4)$

$$\frac{4-1}{8+1} = \frac{3}{9} \quad \frac{1}{3}$$

$$\frac{2}{3}x^{-\frac{1}{3}} = \frac{1}{3} \quad x^{-\frac{1}{3}} = \frac{1}{2}$$

no



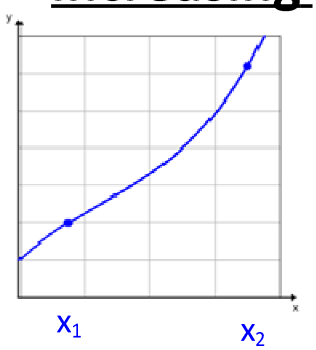
$$\frac{1}{\sqrt[3]{x}} = \frac{1}{2}$$

$$(2)^3 = (\sqrt[3]{x})^3$$

$x=8$

Increasing & Decreasing Functions

Increasing

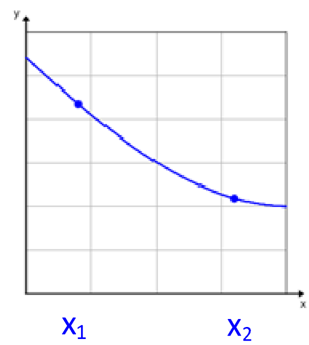


f is increasing if $x_1<x_2$ and $f(x_1)<f(x_2)$

If $f'(x)>0$ at each point in (a,b), then

f increases on (a,b)

Decreasing



f is decreasing if $x_1<x_2$ and $f(x_1)>f(x_2)$

If $f'(x)<0$ at each point in (a,b), then

f decreases on (a,b)

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Ex3. Find all possible functions with the given derivative

1.) $f'(x) = 6x^2 + 2x + 3$

general antiderivative

$f(x) = 2x^3 + x^2 + 3x + C$

2.) $g'(x) = \sec^2 x$

Ex4. Find the function with the given derivative that passes through the given point

1.) $y' = 3e^{3x}$ $(\overset{x}{0}, \overset{y}{4})$

$y = e^{3x} + C$

$4 = e^{3(0)} + C$

$4 = 1 + C$ $C = 3$

$y = e^{3x} + 3$

specific antiderivative

2.) $h'(x) = 2x+5$ $(2,7)$

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Homework

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